Restoration of Faded Slides

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1 Introduction

Colour transparencies using the reversal process do not remain stable over periods of years. There are broadly speaking two photographic processes which behave differently when stored. The Kodachrome K14 process produces slides that are stable over many years when kept in the dark but ultimately acquire a blue cast; they also deteriorate rapidly when exposed to light. Conversely the E4 or E6 process used by Ektachrome and all non-Kodak films is more stable when exposed to light but both in the light and in the dark deteriorates within a few years, usually acquiring a strong pink cast.

These notes attempt to model the physical processes responsible for the colour changes and describe a digital processing algorithm for restoration which is implemented in `gimp`.

2 The Colour Photographic Reversal Process

2.1 The Principle of the Reversal Process

The behaviour of a photographic emulsion when exposed to light and developed is described by the equation

$$D = \gamma_{\mathrm{p}} \log(E/E_0)$$

where E is the exposure in lux-seconds, E_0 and γ_{p} are constants and D is the density of silver deposited. The density is defined by the attenuation of light passing through the film; let the incident intensity of light when viewing the film be P_{i} and the transmitted intensity be P_{t} then

$$D = \log \frac{P_{\mathrm{i}}}{P_{\mathrm{t}}}.$$

Combining these gives

$$P_{\mathrm{t}} = P_{\mathrm{i}} \left(\frac{E}{E_0} \right)^{-\gamma_{\mathrm{p}}}.$$

The experimental value of γ_p is ~ 0.55 but varies a bit with the formulation of the emulsion and the development time.

Let $D_{\max}$ be the maximum value of D which will be determined by the composition of the emulsion and the value will have been chosen so that very little light is transmitted through a film with this density. Suppose this corresponds to an exposure $E_{\max}$, so that

$$D_{\max} = \gamma_p \log(E_{\max}/E_0).$$

In the reversal process the silver is removed and the *undeveloped* emulsion is used which will have a density $D_{\max} - D$. Using the above equations we find

$$D_{\max} - D = \gamma_p \log(E_{\max}/E_0) - \gamma_p \log(E/E_0) = -\gamma_p \log(E/E_{\max}).$$

This undeveloped emulsion is converted to a coloured dye so the amount of dye is given by $\beta(D_{\max} - D)$ where β is some constant. We could define the amount of dye in units of say μg mm^{-2} in order to give a numerical value to β. Let the absorption coefficient of the dye be a, that is the intensity transmitted by a density of 1μ mm^{-2} is

$$P_t = P_i \, e^{-a},$$

and in general a will depend on wavelength.

The intensity of light transmitted by the dyed layer is then given by

$$P_t = P_i \, e^{-a\beta\gamma_p(D_{\max} - D)} = P_i \left(\frac{E}{E_{\max}}\right)^{a\beta\gamma_p}.$$

Presumably the manufacturer has designed the emulsion and processing so that $a\beta\gamma_p \simeq 1$ and the intensity of the transmitted light P_t is approximately proportional to the original exposure E.

2.2 Ideal Colour Photography

In a reversal colour film there are three emulsions sensitive to red, green and blue light and after development there will be silver densities D_r, D_g and D_b. When the slide is viewed the white incident light passes through all three layers and is attenuated by each of them. Let the attenuation coefficients of the dyes at wavelength λ be $a_r(\lambda)$, $a_g(\lambda)$ and $a_b(\lambda)$, then the spectrum of white light transmitted by all three layers is

$$P_t(\lambda) = P_i \times e^{-a_r(\lambda)\beta_r(D_{\max} - D_r)} \times e^{-a_g(\lambda)\beta_g(D_{\max} - D_g)} \times e^{-a_b(\lambda)\beta_b(D_{\max} - D_b)}.$$

We now idealise the situation; assume the wavelength is in the red part of the spectrum and the dye in the red layer absorbs strongly in the red but the dyes in the green and blue transmit red perfectly so that $a_r(\text{red}) \sim a_{\max}$ and $a_g(\text{red}) \simeq 0$ and $a_b(\text{red}) \simeq 0$. This implies that

$$P_{\mathrm{t}}(\text{red}) = P_{\mathrm{i}} \times \mathrm{e}^{-a_{\max}\beta_r(D_{\max}-D_r)} = P_i \times \left(\frac{E_r}{E_{\max}}\right)^{a_{\max}\beta_r\gamma_{\mathrm{p}}} .$$

Assuming $a_{\max}\beta_r\gamma_{\mathrm{p}} = 1$ we get

$$P_{\mathrm{t}}(\text{red}) = P_{\mathrm{i}} \times \left(\frac{E_r}{E_{\max}}\right),$$

where we have defined E_r as the exposure that gave rise to D_r. The intensity of red light emerging from the transparency is therefore proportional to the intensity of red light falling on the film. The same argument applies to the green and blue.

2.3 Scanning and digitisation

We now consider the steps by which a transparency is scanned and recorded as a computer file. The scanner uses a white light source and the light transmitted by a small region of the transparency falls on a detector which measures the intensity in red, green and blue channels. The colour discrimination is presumably done using filters in front of each element of the detector chip and we need briefly to consider these. In contrast to the coloured dyes used in photographs the choice of substance for these filters is not constrained by complex chemistry and they can use 'state of the art' materials. This implies in particular that the 'cross-talk' between colours is small; that is the red detector, for example, is insensitive to green light and completely insensitive to blue light.

The detailed spectrum of the scanner light source and the sensitivities of the different colour detectors need calibration and presumably the scanner does this automatically when it is switched on and the appropriate calculations are done for every pixel in the digitised image. It is worth noting that the spectral sensitivities of the detectors differ significantly from the absorption spectra of the photographic dyes. Presumably this is corrected for in the scanner software.

We will assume that the image is digitised with one byte per pixel for each colour, so intensity values are in the range 0 to 255. These numbers are *not* proportional to the light intensity falling on the detector because the dynamic range of intensities in a typical photograph is much more than 255 to 1. A non-linear transformation is used to compress the intensities; the light intensity transmitted by the transparency P_{t} is related to the integer R in the computer file by

$$P_{\mathrm{t}} = P_{\max}\left(\frac{R}{255}\right)^{\gamma_{\mathrm{d}}},$$

where γ_d is a constant, usually equal to 2.2 . The value used by the scanner is normally embedded in the digitised file so that the transformation can be reversed before the image is displayed. A different constant is used for the display on Mac computers from those running Windows or Linux.

P_max is a constant equal to the light intensity which is converted to digital value 255 and we need to consider how this is fixed because it has an important effect on the restoration algorithm. Experiments reveal that the brightest pixel in an image often has a digital value close to 255 and this suggests that the scanner scales intensities to achieve this. However scanning a uniformly dark image with no bright pixels produces maximum values much less than 255 and clearly the scanner has not scaled these. We conclude that the scanner must record absolute light intensities. Presumably during the calibration on start-up the intensity of the light with no transparency inserted is recorded and this is noted as P_max and digitised as 255.

3 The Simplest Model for Fading

Let c stand for a colour, red, green or blue, then the previous section derived the equation relating the transmitted light to the original exposure:

$$P_t(c) = P_i \times \left(\frac{E_c}{E_\text{max}}\right)^{a_c(c)\beta_c\gamma_\text{p}},$$

and it was assumed that $a_c(c)\beta_c\gamma_\text{p} = 1$. The simplest model for fading is that the quantities of the dyes in the emulsion are reduced during the passage of time. This is quantified by a reduced value of the constant β. In the following primed quantities will refer to the degraded transparency so we have

$$P'_t(c) = P_i \times \left(\frac{E_c}{E_\text{max}}\right)^{a_c(c)\beta'_c\gamma_\text{p}} \quad \propto \quad [P_t(c)]^{\alpha_c},$$

where $\alpha_c = \beta'_c/\beta_c < 1$.

In terms of the numbers C and C' in the computer file

$$P'_\text{max}\left(\frac{C'}{255}\right)^{\gamma_\text{d}} = P_\text{max}\left(\frac{C}{255}\right)^{\alpha_c\gamma_\text{d}}.$$

P'_max refers to the scanning of the degraded transparency and P_max to the corresponding value had it been possible to scan the original. We argued above that these numbers must be fixed by the scanner and are independent of the slide, so they are equal. This gives the simple formula

$$C' = 255 \times \left(\frac{C}{255}\right)^{\alpha_c}$$

and the restoration formula

$$C = 255 \times \left(\frac{C'}{255}\right)^{1/\alpha_c}.$$

4 More Sophisticated Models

4.1 Cross-Colour interference ('Side absorptions')

We return to the formula

$$P_{\mathrm{t}}(\lambda) = P_{\mathrm{i}} \times \mathrm{e}^{-a_r(\lambda)\beta_r(D_{\max}-D_r)} \times \mathrm{e}^{-a_g(\lambda)\beta_g(D_{\max}-D_g)} \times \mathrm{e}^{-a_b(\lambda)\beta_b(D_{\max}-D_b)}$$

for the intensity transmitted through all three layers of the emulsion where earlier we assumed $a_r(\text{red}) = a_{\max}$, $a_g(\text{red}) = a_b(\text{red}) = 0$. The dyes in the green and blue layer are magenta and yellow and are supposed to transmit red without absorption. Let us suppose that in practice they do absorb some red, so we will remove this simplification. This leads to

$$P_{\mathrm{t}}(\text{red}) = P_{\mathrm{i}} \times \mathrm{e}^{-a_r(\text{red})\beta_r(D_{\max}-D_r)} \times \mathrm{e}^{-a_g(\text{red})\beta_g(D_{\max}-D_g)} \times \mathrm{e}^{-a_b(\text{red})\beta_b(D_{\max}-D_b)}$$

$$= P_{\mathrm{i}} \times \left(\frac{E_r}{E_{\max}}\right)^{a_r(\text{red})\beta_r\gamma_{\mathrm{p}}} \times \left(\frac{E_g}{E_{\max}}\right)^{a_g(\text{red})\beta_g\gamma_{\mathrm{p}}} \times \left(\frac{E_b}{E_{\max}}\right)^{a_b(\text{red})\beta_b\gamma_{\mathrm{p}}} .$$

When the dyes deteriorate each β is replaced by $\beta' = \alpha\beta$. Assuming $a_r(\text{red})\beta_r\gamma_{\mathrm{p}} = a_g(\text{green})\beta_g\gamma_{\mathrm{p}} = a_b(\text{blue})\beta_b\gamma_{\mathrm{p}} = 1$ this can be written

$$P_{\mathrm{t}}(\text{red}) = P_{\mathrm{i}} \times \left(\frac{E_r}{E_{\max}}\right)^{\alpha_r} \times \left(\frac{E_g}{E_{\max}}\right)^{\alpha_g\delta_{rg}} \times \left(\frac{E_b}{E_{\max}}\right)^{\alpha_b\delta_{rb}}$$

where $\delta_{rg} = a_g(\text{red})/a_g(\text{green})$ and $\delta_{rb} = a_b(\text{red})/a_b(\text{blue})$. The implications of this formula are that even with no degradation ($\alpha = 1$) the colours are slightly mixed up and deterioration in the dyes in the green and blue layers affects the intensity of the red. This has no effect in bright areas ($E_g \sim E_{\max}$ and $E_b \sim E_{\max}$) but will affect the colour balance in the dark areas. The effect is small because δ is small and although it is discernible, particularly affecting the blue-green balance, it is not enough to prevent effective colour photography. It may become important in a faded transparency because the three numbers α_r, α_g and α_b may be very unequal; for example, if $\alpha_r \ll 1$ and $\alpha_g \simeq \alpha_b \simeq 1$ the mixing factors become significant.

4.2 Further Colour Production

In the K14 process the only chemicals left in the emulsion are the dyes themselves. The most plausible mechanism for deterioration in these emulsions is that the dye molecules are destroyed and the emulsion becomes more transparent. It is theoretically possible for the dye molecules and their corresponding absorption spectra to be modified. It is also possible for some other part of the emulsion such as the film base, which was intended to be completely transparent, has become coloured with age. A change in the dye spectrum can be modelled by a loss of the original dye plus additional absorptions so both mechanisms can be considered together.

In contrast, in the E4 and E6 process it is the oxidised developer that reacts with 'colour-couplers' in the emulsion to produce the dyes and any unreacted 'colour-couplers' are left in the emulsion. Therefore, in addition to the ageing processes occurring in the K14 process it is theoretically possible for more dye to be produced later. Assuming sufficient colour-coupler remains the amount of extra dye could be the same all over the emulsion, or a fraction of the remaining colour-coupler could be converted to dye. In this case the extra dye at a particular point will depend on the exposure there.

We analyse these two cases. Let the original amount of colour-coupler in the emulsion be $d_{\max}$, the amount of dye produced is $\beta(D_{\max} - D)$ and the amount of coupler remaining $d_{\max} - \beta(D_{\max} - D)$. Presumably the manufacturer designs the emulsion so that $d_{\max} \simeq \beta D_{\max}$ since then if $D = 0$, that is no exposure, all the coupler is used up. There is no point in making it any larger. It follows that the amount of unused coupler is βD.

Starting from the equation for the transmitted light intensity:

$$P'_{\mathrm{t}}(\lambda) = P_{\mathrm{i}} \times \mathrm{e}^{-a_r(\lambda)\beta'_r(D_{\max}-D_r)} \times \mathrm{e}^{-a_g(\lambda)\beta'_g(D_{\max}-D_g)} \times \mathrm{e}^{-a_b(\lambda)\beta'_b(D_{\max}-D_b)},$$

assuming the extra dye density is constant the effect on the first factor is to change it to

$$\mathrm{e}^{-a_r(\lambda)[\beta'_r(D_{\max}-D_r)+\beta_r\gamma_{\mathrm{p}}\eta_r]},$$

where the extra dye density is described by η_r and the factors $\beta_r\gamma_{\mathrm{p}}$ are included for convenience. If the extra dye is a fraction θ_r of the unused coupler we get

$$\mathrm{e}^{-a_r(\lambda)[\beta'_r(D_{\max}-D_r)+\theta_r\beta_r D_r]}.$$

Taking λ in the red and $a_r(\mathrm{red})\beta_r\gamma_{\mathrm{p}} = 1$ the first of these gives

$$\left(\frac{E_r}{E_{\max}}\right)^{\alpha_r} \mathrm{e}^{-\eta_r}.$$

Under the same assumptions the second becomes

$$\left(\frac{E_r}{E_{\max}}\right)^{\alpha_r} \left(\frac{E_r}{E_0}\right)^{\theta_r} = \left(\frac{E_r}{E_{\max}}\right)^{\alpha_r-\theta_r} \left(\frac{E_{\max}}{E_0}\right)^{\theta_r}.$$

Both these models therefore lead to the same mathematical form for the factor, namely a power and a scaling. We can also write α_r in place of $\alpha_r - \theta_r$ since the value depends only on the degree of degradation of the dyes which is not known.

4.3 Final Results

Following the digitisation process as described previously we can write equations relating the observed intensities in the red, green and blue (R', G', B')

to the original ones (R, G, B)

$$
\begin{array}{llllll}
R' = 255F_r & \times & \left(\frac{R}{255}\right)^{\alpha_r} & \times & \left(\frac{G}{255}\right)^{\alpha_g\delta_{gr}} & \times & \left(\frac{B}{255}\right)^{\alpha_b\delta_{br}} \\
G' = 255F_g & \times & \left(\frac{R}{255}\right)^{\alpha_r\delta_{rg}} & \times & \left(\frac{G}{255}\right)^{\alpha_g} & \times & \left(\frac{B}{255}\right)^{\alpha_b\delta_{bg}} \\
B' = 255F_b & \times & \left(\frac{R}{255}\right)^{\alpha_r\delta_{rb}} & \times & \left(\frac{G}{255}\right)^{\alpha_g\delta_{gb}} & \times & \left(\frac{B}{255}\right)^{\alpha_b}
\end{array}
$$

The scaling factors for the three emulsions are simply multiplied; if there is absorption at a particular wavelength it is immaterial in which layer it occurs and we have written F_r, F_g, and F_b for the overall extra absorptions.

5 The Restoration Algorithm

5.1 Overview

The equations at the end of the previous section describe the model of degradation of the dyes in the emulsion. The constants δ are known approximately from published absorption spectra of the dyes; but the values of α_r, α_g, α_b and F_r, F_g, F_b have to be estimated from the degraded slide. This is in general impossible without an interpretation of the picture and the best results will inevitably involve human judgement. It is worth stressing however that even if the restoration is done interactively the theoretical above analysis strongly restricts the operations that should be needed. It should be possible to accomplish the restoration using *only* scaling and power law operations. The latter are usually called γ-adjustments. The `gimp` procedure `levels` does both of these.

The intention of the current work is to design an algorithm that estimates the six parameters α and F automatically and produces an acceptable restoration for a 'typical' picture. Having got close to the optimum restored image only small interactive adjustments should be needed.

The algorithm involves three stages (1) a crude restoration getting the dye densities to approximately the right values, (2) a colour balance making small changes to reduce the overall colour cast, and (3) a correction for the side-absorptions.

5.2 An Example

The original of first example is shown below in figure 3. This is a typical Agfacolour slide scanned after being stored for several years[1]. The lack of dark colours and the overall pink cast suggest a serious loss of cyan dye. In order to confirm this we can look at the histograms of the distributions of red green and blue pixels. A procedure for computing these is part of most photo-processing packages, however this computes the histograms for the *pixels* in the image and the results will depend on the number of pixels

[1] I am grateful to Tim Gossling for supplying this

of each colour. If, for example, there is a large area of blue sky the blue histogram will be skewed towards high values. We are not interested in this aspect of the image content but only in the colours that occur and these can conveniently be described by converting the image to `colour-indexed` mode. There is a `gimp` procedure for this which attempts to represent the image using a only prescribed number of colours which will be referred to as the 'colour palette' of the image. If we now examine the histograms of these colours we can get quantitative measures of the colours present.

Figure 1 shows the red green and blue histograms of the colour distributions for the first example. These were computed for the optimum 256 colours and counted in 16 bins. The empirical conclusions about the image are con-

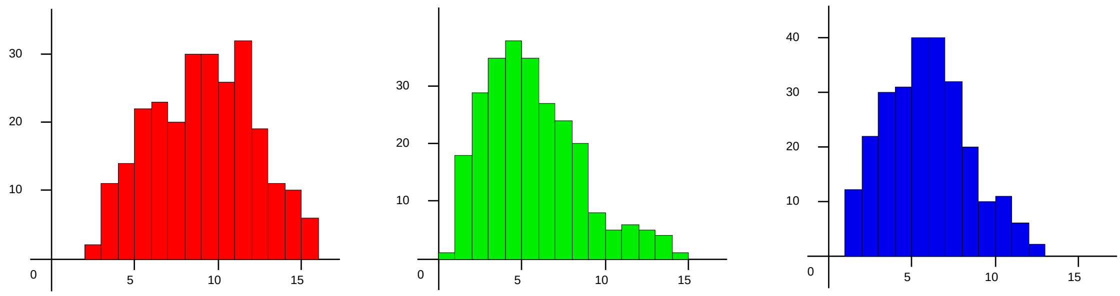

Figure 1: Histograms of the optimum 256 colours to represent the image in figure 3

firmed; there are no low intensity red values and a lot of high intensity ones. The average in the red channel is also high. This is what one would expect for a loss of cyan dye. In contrast there are some dark green colours and some bright ones but a striking feature of the blue channel is the complete lack of high values which must imply an excess of yellow dye.

5.3 The Crude Restoration

At this stage we ignore the side absorptions so the colours are treated independently and the formula relating the degraded red colour value to the original is

$$R' = m_r \left(\frac{R}{255}\right)^{\alpha_r} \quad \text{or} \quad R'' = 255 \left(\frac{R'}{m_r}\right)^{1/\alpha_r},$$

with similar formulas for green and blue. Here we have written R'' for the restored value of R' which we hope will approximate R. We have also defined the scaling by m_r in place of F_r and we can see that $R' = m_r$ corresponds to $R'' = 255$.

We would expect the histograms of the red, green and blue colours in the colour palette of the indexed image to be similar and we must choose the scaling and γ parameters to achieve this.

A common assumption in photo-processing software is that there are some white pixels somewhere in the image and hence white should be an entry in the colour palette. Since γ adjustment does not affect white the scaling parameters m_r, m_g and m_b can be fixed to the maximum (R, G, B) values in the palette so these will all be moved to 255.

We make the assumption that in a good photograph the colours in the optimum palette will be spread uniformly over the intensity range. We therefore use the α parameters to stretch the histograms so that the whole intensity range is used in each colour. In order to design a more robust algorithm we move the lower decile of intensities rather than the lowest value.

The procedure that computes the optimum colour palette can be thought of as a mathematical function which converts a set of (R, G, B) values to a set of colours denoted by (r, g, b):

$$\{(r, g, b)\} = \Phi\big(\{(R, G, B)\}\big),$$

where curly brackets $\{\cdots\}$ are used to denote a set. We wish to determine parameters α and m such that the restored pixels $R'' = 255(R'/m_r)^{1/\alpha_r}$, and similarly G'' and B'', produce a palette $\Phi\big(\{(R'', G'', B'')\}\big)$ whose lower deciles occupy one tenth of the possible range for each colour.

In the optimum palette a single colour is used in place of a set of colours occupying a small volume of RGB space. Therefore the palette entry (r, g, b) used to replace a colour pixel (R, G, B) is approximately given by $(r, g, b) \simeq (R, G, B)$. It follows that the red palette entry for (R'', G'', B'') is approximately $255(r/m_r)^{1/\alpha_r}$. We can use this to get an approximate value of α_r. We compute the low decile ℓ_r of the numbers $255(r/m_r)^{1/\alpha_r}$ and equate this to the the suggested value $255 f_{\text{low}}$. For generality the number f_{low} is written in place of the actual value 0.1. This gives

$$f_{\text{low}} = \left(\frac{\ell_r}{m_r}\right)^{1/\alpha_r},$$

which gives an approximate value of α_r:

$$\alpha_r = \frac{\log(\ell_r) - \log(m_r)}{\log(f_{\text{low}})}.$$

Iterating the palette This method of estimating the parameter α is based on applying the 'restoration' to the colours in the palette rather than restoring the image and recomputing the optimum palette. The method is approximate because an entry in the optimum palette is representative of the colours occupying a small volume of RGB space. When the colours are 'restored' these volumes are distorted, so although the restored colours are sensible ones to represent the restored image they are not optimal.

We therefore iterate the process, changing the parameters α and m until the lower decile of $\Phi\big(\{(R'', G'', B'')\}\big)$ equals $255 f_{\text{low}}$. We do this using a method similar to the Newton-Raphson algorithm. We compute the function Φ by restoring the image and recomputing the optimum palette. The derivative of the function Φ with respect to α is obtained from the approximate result used above

$$\Phi\big(\{((R/m)^{1/\alpha}, G, B)\}\big) \simeq \{(255(r/m)^{1/\alpha}, g, b)\}$$

Since doing the scaling and γ-adjustment and computing the optimum colour palette are very slow operations we do them on a smaller image of one tenth of the size. The optimum colours are almost identical to those required for the full image.

The values for Example 1 are:

	red	green	blue
maximum	251	224	193
lower decile	77	37	42
initial α	0.513	0.782	0.662
final m	249	222	188
final α	0.457	0.716	0.589

Table 1: Parameters used for Restoration

The computed values are shown in table 1 and the extent of the dye loss can be seen to be more than half the cyan dye and significant amounts of the magenta and yellow.

5.4 The Colour Balance

Another way of looking at the distribution of colours is to plot the entries in the palette against the index. This depends on how the palette entries are ordered so such a plot has no absolute significance but it is useful to examine the overall colour balance in the image. Such a plot for the palette of the crudely restored Example 1 is shown on the left in figure 2.

The low values at the left are much below the previous values in the table above and the maximum values are now all 255. In spite of fact that the three curves are similar at the low and high ends it is clear that the green curve lies below the red and blue. This means that the majority of colours used to represent the image still have a magenta tinge. Also for some ranges of intensity blue is slightly above red and for some slightly below. These predictions are noticeable in the crudely restored image (not illustrated).

It is clear that our method of fixing the α parameters independently is not entirely satisfactory although it gets near to sensible values. We therefore do a refinement of them to bring the three curves closer together. This should ensure that on average no colour can predominate.

Let define an 'average colour' for index i by $A_i = (R_i'' + G_i'' + B_i'')/3$ and suppose we replace our restored colours by $R''' = \sigma_r R''^{\lambda_r}$ and similarly for

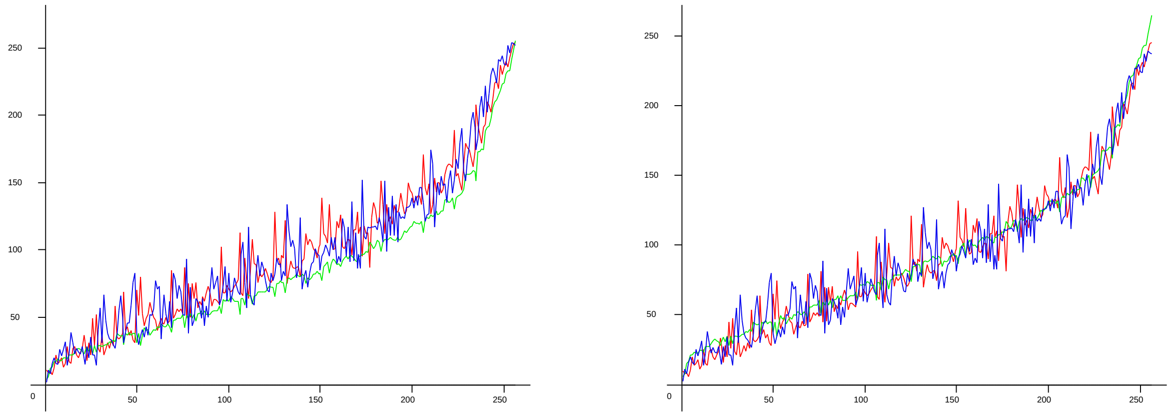

Figure 2: The colours in the optimum palette, left: after crude restoration, right: after colour balancing.

G and B. We now choose parameters σ and λ to minimise the sum of the mean square differences between A and each of R''', G''' and B''', that is minimise

$$\sum_i \left(A_i - \sigma_r R_i''^{\lambda_r}\right)^2 + \left(A_i - \sigma_g G_i''^{\lambda_g}\right)^2 + \left(A_i - \sigma_b B_i''^{\lambda_b}\right)^2$$

Several comments are in order. We have allowed a change in the scaling parameter (introduced σ) and hence abandoned the principle that the brightest pixel in the restored image should be white. Experiments showed that the improved restorations that resulted justified the small changes in the white colour. It would have been easier to determine σ and λ by fitting to the logarithms of A but because of the non-linearity of the logarithm function the differences between the colours were not completely removed and an overall bias was still noticeable. It is also worth noting that the numbers R, G and B are not the brightness in the image because of the γ_d compression. Tests showed that fitting the decompressed data was not, on the whole, an improvement.

Differentiating the above expression leads to a set of non-linear equations for the parameters σ and λ which are solved by an iterative algorithm based on the Newton-Raphson method. The resulting colours are shown on the right in figure 2. The resulting parameters can be combined with the ones from the crude restoration to produce a single scaling and γ correction.

5.5 The Side Absorptions

The exact equations relating the observed values (R', G', B') to the originals was given above. In order to simplify the notation define $\mathcal{R}' = R'/(255F_r)$

and $\mathcal{R} = R/255$ and similarly for the green and blue values. Then

$$\begin{aligned}
\mathcal{R}' &= \mathcal{R}^{\alpha_r} && \mathcal{G}^{\alpha_g \delta_{gr}} && \mathcal{B}^{\alpha_b \delta_{br}} \\
\mathcal{G}' &= \mathcal{R}^{\alpha_r \delta_{rg}} && \mathcal{G}^{\alpha_g} && \mathcal{B}^{\alpha_b \delta_{bg}} \\
\mathcal{B}' &= \mathcal{R}^{\alpha_r \delta_{rb}} && \mathcal{G}^{\alpha_g \delta_{gb}} && \mathcal{B}^{\alpha_b}
\end{aligned}$$

The exact solution of these equations can be obtained by writing, for example, $\mathcal{R} = \mathcal{R}'^x \; \mathcal{G}'^y \; \mathcal{B}'^z$ and equating powers of $\mathcal{R}$, $\mathcal{G}$ and $\mathcal{B}$ to obtain x, y and z. However since all the δ's are small we can immediately write down an approximate solution involving the restored values $\mathcal{R}''$, $\mathcal{G}''$ and $\mathcal{B}''$:

$$\begin{aligned}
\mathcal{R} &= \mathcal{R}'' && \mathcal{G}''^{-\alpha_g \delta_{gr}/\alpha_r} && \mathcal{B}''^{-\alpha_b \delta_{br}/\alpha_r} \\
\mathcal{G} &= \mathcal{R}''^{-\alpha_r \delta_{rg}/\alpha_g} && \mathcal{G}'' && \mathcal{B}''^{-\alpha_b \delta_{bg}/\alpha_g} \\
\mathcal{B} &= \mathcal{R}''^{-\alpha_r \delta_{rb}/\alpha_b} && \mathcal{G}''^{-\alpha_g \delta_{gb}/\alpha_b} && \mathcal{B}''
\end{aligned}$$

It is debatable whether it is better to take out only the effects of the fading and restore to the original photograph or to correct for the side-absorptions in the original as well, with the intention of getting closer to the original scene. We have chosen to do the latter.

There are technical issues in implementing the above equations in `gimp`. If logarithm and exponential functions were implemented the operations could be done using the `channel-mixer` plug-in although there would be accuracy issues with only 8 bit arithmetic. Without such functions we could use the `levels` procedure to compute the necessary powers however there are limits on the γ value permitted in this procedure which rule out very small or negative exponents. It is also possible for the corrections to be so large for dark colours that imperfections in the digitisation and restoration spoil the result. We have therefore adopted the simple approach of using the approximation $a^x = \exp(x \log a) \simeq 1 + x \log a$ which is valid for small x and then assuming $a \sim 1$ so $\log a \simeq -(1-a)$ giving $a^x \simeq 1 - x(1-a)$. This gives a similar result to the exact one for bright colours but under-corrects the dark ones. This can now be implemented using the `channel-mixer` and `invert` functions to create an extra layer which is divided into the image.

5.6 Empirical Correction

The results following the restoration algorithm described are generally acceptable but the overall colours tend to be colder than one would produce interactively. The reason for this is probably that the colour balance produced by the algorithm which attempts to get the red, green and blue values equal, is not what appears most attractive to the eye. An overall shift to from blue to yellow using the `color-balance` procedure has been added. This improves many pictures and none appears to be made worse by this step.

6 Results

The first example is the case discussed in detail above and shown on the left in figure 3. This presents a very challenging problem, there is severe dye loss, the contrast range is very high with the white shirt in the foreground and deep shadows, and the range of colours present is unusual. The picture contains no sky, almost no green objects, and large regions of ground and the table whose grayish colour will be fixed by the colour-balancing phase.

The restored result on the right still shows a slight magenta tinge and is still rather cold.

The second picture in figure 4 is a good example of a cheap colour film using the E4/E6 process. The original was clearly taken in poor light on a wet day (note the car headlights) but very serious subsequent deterioration has occurred. In the circumstances the restoration is surprisingly good.

Example 3 (figure 5) is a Kodachrome picture about 35 years old. It shows fading and the development with age of a blue cast colour that is typical of the K14 process. Again the restored version is acceptable.

7 Conclusions

These document has described an algorithm for the restoration of transparencies that have deteriorated with age due to loss of the coloured dyes in the emulsion. It is designed to do this automatically, estimating the amount of dye loss from the picture and correcting for it. No intervention by the user is needed although in the current implementation there are options to remove the yellow shift and to reduce the contrast. In many cases it produces acceptable results from unacceptable originals, in others it gets sufficiently close to an acceptable solution that only a simple final correction by the human user is necessary.

The algorithm is implemented as a `gimp plug-in` written in python.

Figure 3: Example 1: Left: Original, Right: Restored

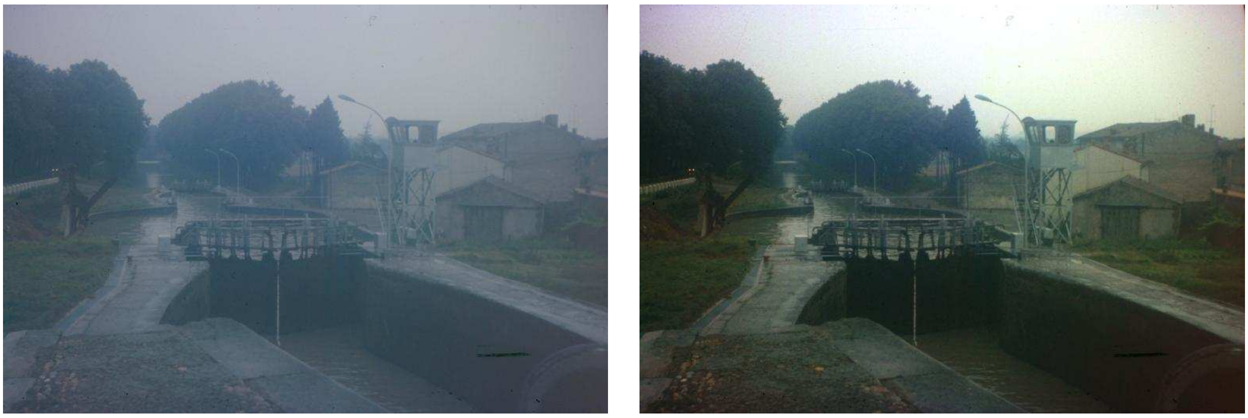

Figure 4: Example 2: Left: Original, Right: Restored

Figure 5: Example 3: Left: Original, Right: Restored